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Phase Transitions in the Early Universe and their Consequences [and Discussion]

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Phase transitions in the early Universe and their consequences

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The reasons for believing that a number of phase transitions occurred in the early Universe are reviewed, and their implications discussed. In particular, the current status of the explanation for the observed values of some constants in terms of the 'inflationary universe' is examined.

Introduction

The observed cosmic redshift and microwave background provide good evidence that at earlier times the Universe was denser and hotter than it now is. Beyond a redshift Z of 1300, corresponding to a temperature of 4000 K, it was filled with a dense plasma of ionized hydrogen, more or less in thermal equilibrium though expanding adiabatically.

At yet earlier times, $t \lesssim 1$ s, when thermal energies exceeded 1 MeV, elementary particle processes dominated. The relevant equation of state then is more speculative. However, if our present ideas about fundamental particle interactions are on the right lines, then it is in fact a reasonable approximation over much of the earlier period to treat the matter in the Universe as a weakly-interacting relativistic gas. In particular, if the strong interactions are indeed describable by quantum chromodynamics (QCD) then asymptotic freedom suggests that at sufficiently high temperature all particle interactions are effectively weak.

Phase transitions

We may additionally expect the smooth adiabatic expansion to be interrupted by a number of phase transitions. (For a discussion and earlier references see Kibble (1980) or Kibble (1982).) The currently accepted theory of electromagnetic and weak interactions – the Weinberg–Salam model – predicts a phase transition at a temperature of order 100 GeV at which the $SU(2) \times U(1)$ symmetry breaks spontaneously to the U(1) of electromagnetic gauge invariance. Moreover in QCD there is almost certainly a phase transition in the vicinity of 100 MeV at which confinement sets in. Above this temperature we have a gas of unconfined quarks and gluons; below, a gas of hadrons.

We can be fairly confident about these two transitions. Both may be accompanied by interesting phenomena of various kinds, but neither is likely to introduce any really dramatic disturbance to the smooth evolution of the Universe. What happens at yet higher temperatures is far more problematic, because we really know very little about particle interactions beyond 1 TeV. However there are good reasons for supposing that other phase transitions occurred.

Since both strong and electroweak interactions are well described by gauge theories, the idea of grand unification (Georgi & Glashow 1974) seems very appealing. In a grand unified theory (GUT) there is a phase transition (or perhaps several) at a temperature T_c of, typically, about 10^{15} GeV. Above this, the equilibrium state is fully symmetric under a group such as SU(5) or SO(10), while at lower temperatures the symmetry breaks to $SU(3)_{colour} \times [SU(2) \times U(1)]_{WS}$.

Unlike the later transitions, this one is probably strongly first-order (Guth & Tye 1980; Lazarides & Shafi 1980; Cook & Mahanthappa 1981; Guth & Weinberg 1981; Einhorn & Sato 1981; Billoire & Tamvakis 1981). We may expect the Universe to supercool in its (metastable) symmetric phase. The equilibrium states correspond to minima of the effective potential or free-energy density $V(\phi)$, where ϕ is some order parameter or Higgs field. In this case, V has two minima separated by a barrier. Below T_c the minimum at $\phi = 0$ is no longer the absolute minimum, but the Universe remains there until it can tunnel through the barrier. During this supercooling the energy density comes to be dominated by the vacuum energy, $V(\phi)$, which is of order T_c^4. (The zero is chosen to be at the absolute minimum of V.) This leads to an exponential expansion,

$$R(t) \propto e^{Ht}, \quad \text{where } H \text{ is of order } T_c^2/M_P. \quad (M_P \text{ is the Planck mass}).$$

This is the *inflationary universe* (Guth 1981).

If the transition is terminated by nucleation of bubbles of the new phase, one finds essentially all the energy concentrated in the bubble walls and hence an impossibly inhomogeneous distribution of matter (Einhorn *et al.* 1980; Guth & Weinberg 1981; Hawking *et al.* 1982). This problem has been at least partly overcome in the *new inflationary universe* (Linde 1982*a*; Albrecht & Steinhardt 1982; Hawking & Moss 1982). In this model, $V(\phi)$ is extremely flat near $\phi = 0$, with no ϕ^2 term. Consequently, even after the transition, ϕ will take a long time to 'roll down' the curve of V towards its absolute minimum. This has two effects. First, exponential expansion will continue for a long time after the transition; thus the bubble size will expand by an enormous factor, so that our present Universe is entirely contained within a single bubble. Second, the energy will not be deposited in the bubble wall but will spread much more uniformly, suggesting that we might avoid the problem of excessive inhomogeneity.

As we shall see, there are still problems with this scenario. In particular, the condition that the potential be very flat requires an extremely precise fine tuning of parameters. However it has been suggested by several authors (Ellis *et al.* 1982; Albrecht *et al.* 1982; Vayonakis 1983) that inflation may occur naturally in a supersymmetric model. There are also difficulties, to be discussed later, concerning the magnitude of the fluctuations in the new inflationary universe.

Another problem that has not been fully resolved concerns the gravitational effects on the phase transition. During the period of exponential expansion the Universe is effectively in a de Sitter space, and once the temperature falls to the associated Hawking temperature T_H, of order H, the curvature corrections to the effective potential become important (Abbott 1981; Hut & Klinkhamer 1981; see also Shore 1980; Pollock & Calvani 1982; Brandenberger & Khan 1982).

Density of the early Universe

Let us concentrate on the Universe just before the electroweak phase transition, when it is say 1 ps old and in thermal equilibrium at a temperature of about 1 TeV. From that point on, we can follow its evolution reasonably well. What we have to understand, therefore, is the initial state at that time.

Let us consider the various parameters that will need to be specified. From the observed isotropy of the microwave background, which is isotropic to better than one part in 10^3, we may conclude

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that the Universe must have been to a good approximation homogeneous and isotropic. Thus it may be represented at least approximately by one of the Robertson–Walker metrics

$$ds^2 = dt^2 - R^2(t)\, d\sigma^2, \tag{1}$$

where $d\sigma^2$ is a 3-metric of uniform curvature K.

The rate of expansion $\dot{R}$, or the Hubble parameter $H = \dot{R}/R$, is related to the density ρ by Einstein's equation

$$H^2 = (\dot{R}/R)^2 = (8\pi/3M_P^2)\,\rho - K/R^2 - \Lambda, \tag{2}$$

where Λ is the cosmological constant and $M_P = G^{-\frac{1}{2}}$ is the Planck mass. Observationally, Λ is very small, consistent with zero:

$$\Lambda/M_P^2 \lesssim 10^{-122}. \tag{3}$$

It is for this reason that the absolute minimum of the effective potential V is taken to be at zero. This may find a natural explanation within the context of a supergravity model, where the energy is positive definite (Deser & Teitelboim 1977), and radiative corrections to Λ may be expected to cancel. However it is very hard to see how such a result would survive supersymmetry breaking. One possible explanation will be discussed in the following paper (Hawking 1983).

If $\Lambda = 0$, the critical density ρ_c that separates closed and open universes is given by

$$\rho_c = 3M_P^2 H^2/8\pi.$$

If $\rho \leqslant \rho_c$ then $K \leqslant 0$; the Universe is open and will continue expanding for ever. If $\rho > \rho_c$, then $K > 0$. In that event the Universe is closed; it will reach a maximum size and then start to recontract.

As the Universe evolves, ρ varies initially like R^{-4}, while it is dominated by relativistic particles, and eventually like R^{-3} when rest-mass is the largest contribution. On the other hand, from Einstein's equation $\rho - \rho_c$ varies much more slowly, like R^{-2}. In the Universe today, ρ is within about an order of magnitude of ρ_c. To achieve this, it must have been much closer in the early Universe. In fact at the time we have chosen

$$|\rho - \rho_c|/\rho \lesssim 10^{-26}; \tag{4}$$

this is another parameter we have to understand.

One of the major successes of the new inflationary universe is that it provides a natural explanation for this very small number. For, during the period of exponential expansion ρ is essentially constant (equal to $V(0)$) while as before $\rho - \rho_c \propto R^{-2}$. In this situation, ρ_c is driven towards, rather than away from, ρ. It is easy to arrange that during inflation R increases by 10^{28} or more, yielding a very adequate reduction in $\rho - \rho_c$.

Baryon and lepton numbers

To specify a thermal equilibrium state we need not only the temperature or density but also the values of any absolutely conserved quantum numbers. So far as physics below 1 TeV is concerned, the only such quantities we known of are electric charge Q, baryon number B and the various lepton numbers L_e, L_μ, L_τ, though of course it is conceivable that there might be further families of leptons.

A homogeneous state of nonzero electric charge is impossible by Gauss's law, and we may therefore assume that on average Q is zero. On the other hand the net baryon number is not zero, certainly in our galaxy, and most likely in the Universe as a whole. It is most easily characterized by the baryon-to-photon ratio which is approximately constant during adiabatic expansion, or better still the baryon-to-entropy ratio n_b/s. Here n_b is the baryon-number density, and s is the entropy density, given to a good approximation by

$$s = (2\pi^2/45)\, N_*\, T^3,$$

where N_* is the effective number of species of massless particles (i.e. helicity states of bosons, plus those of fermions times $\frac{7}{8}$). This ratio has the advantage that it remains constant even during the various annihilation episodes when particle pairs annihilate and enhance the photon number, provided the processes are reversible.

From observations of matter in the Universe today, we find that

$$n_b/s \approx 10^{-11\pm1}. \tag{5}$$

This again is a parameter that needs explanation.

The lepton numbers are far less accurately known. It is not even wholly inconceivable that the ratio n_l/s could be of order 1, corresponding to a degenerate sea of neutrinos of some type (Langacker *et al.* 1982). The three corresponding lepton-to-entropy ratios complete the list of equilibrium parameters.

A period of inflation naturally leads to essentially zero values of all conserved quantities. Whatever their pre-existing values, they are diluted by an enormous factor.

The only parameter that is definitely non-zero is the baryon-to-entropy ratio. It was one of the first great successes of the GUT idea that it provided a natural explanation for this non-zero value.

In the high-temperature symmetric phase of GUTS, baryon-number-violating processes occur quite freely. This is because at a fundamental level the quarks and leptons belong to the same multiplet. Transitions between them (which induce for example proton decay) are mediated by the exchange of superheavy X bosons. Since m_X is of order 10^{15} GeV, the transition rates are heavily suppressed except at very high temperatures.

We may imagine therefore that at very early times the Universe was in this high-temperature symmetric phase and that the observed non-zero mean baryon number was generated by irreversible CP-violating processes following the phase transition. These could be decay of X bosons or of the associated Higgs bosons, but there are other possibilities too: for example, the decay of Higgs-field fluctuations produced at a first-order transition (Hawking & Moss 1983; see also, Abbott *et al.* 1982; Dolgov & Linde 1982), the annihilation of monopoles or the decay of string loops (Bhattacharjee *et al.* 1982).

Although this explanation for a non-zero baryon number was, and indeed still is, one of the great successes of grand unification, it is important to recognize that it is a very partial success. It provides a qualitative understanding, but at present it cannot provide a quantitative estimate. All it can do is to relate one small parameter, the baryon-to-entropy ratio, to another, the fundamental CP-violation parameter ϵ. At present we have no means of calculating this parameter *a priori*.

Moreover many grand unified theories do not fully satisfy the so-called gauge principle, in that there remain some exact conservation laws corresponding to global invariances, for example a fermion number equivalent to conservation of $B-L$, or indeed separate fermion numbers for different generations. It is then an *assumption* that the mean value of this quantum number is zero in the intial symmetric phase.

Once again of course the new inflationary universe provides a natural explanation for the zero value of any such quantum number. It therefore does make a definite prediction that the lepton-to-entropy ratio should be equal to the baryon-to-entropy ratio. This is a distinctly non-trivial prediction that should in principle be testable. Moreover there are other predictions concerned with fluctuations to which I shall return.

Defects in the initial state

So far we have discussed only equilibrium parameters, but we know that the early Universe cannot have been precisely in a homogeneous equilibrium state. The statistical fluctuations in such a state would be far too small to yield the distribution of mater that we see today.

To complete the description of the state of the Universe at 1 ps we have to specify what inhomogeneities it contains and how it departs from the simple Robertson–Walker form. The inhomogeneities include both spatial fluctuations in the equilibrium parameters (and the metric) and also topological defects of various sorts.

Let us consider first the defects. Depending on the topology of the gauge group and its unbroken subgroup, a phase transition may generate defects of various spatial dimensions: monopoles, strings or domain walls. Monopoles appear naturally in almost all GUTs, while strings and domain walls appear in only certain models (Kibble 1976). In the case of multiple phase transitions, there are also composite structures, strings terminated by monopoles or walls bounded by strings (Bais 1981; Kibble *et al.* 1982*a*, *b*). However these objects usually disappear quite quickly, and for simplicity I shall ignore them. I shall confine my discussion to topologically *stable* defects.

Domain walls within our presently visible Universe can be ruled out because their gravitational effects would induce enormous anisotropy (Zel'dovich *et al.* 1974). Until recently this argument has been used to exclude theories with stable domain walls corresponding to the breaking of an exact discrete symmetry. However the inflationary universe may rehabilitate such models, because any domain walls would probably be so far distant as to be undetectable.

Monopoles like baryons are best characterized by the monopole-to-entropy ratio n_m/s. The requirement that monopoles now contribute to the mean density of the universe no more than about the critical density ρ_c implies that at present $n_m/s \lesssim 10^{-24}$. Since very little annihilation can have occurred after our chosen initial time of 1 ps, this limit must apply then too (Preskill 1979; Zel'dovich & Khlopov 1978; Goldman *et al.* 1981). Another even stronger limit, $n_m/s \lesssim 10^{-25}$, is based on the survival of the galactic magnetic field (Turner *et al.* 1982, Lazarides *et al.* 1981; Arons & Blandford 1983). The new inflationary universe naturally predicts for monopoles too an essentially zero value, thus solving the 'cosmic monopole problem' (Guth 1981). Indeed if the reported detection of a monopole (Cabrera 1982) is confirmed, we should have to suppose that monopoles, like baryons, are made after the main period of inflation, for example at a subsequent monopole-generating phase transition. At first sight one might suppose that it would then again be very difficult to avoid an excess of monopoles. However, Moss (1983) has recently shown that this may not be so, in the context of an SU(5) model with a two-stage transition via SU(4) × U(1) to SU(3) × SU(2) × U(1).

There are other suggestions for reducing, the monopole density, for example the model of Langacker & Pi (1980) involving an intermediate phase where the monopoles disappear. But E. Weinberg (1983) has recently shown using very general causality arguments that all such models are perilously close to the observational limits.

Strings too would be reduced to essentially zero density by inflation. If they are to have any relevance to observational cosmology, they must be produced at a subsequent transition. However it is not at all difficult to meet the observational limits in this case.

As before, we may impose the requirement that their total contribution to the mean density of the Universe can at no time much exceed the critical density. Indeed, in recent times (i.e. since decoupling) it must have been some orders of magnitude less to avoid introducing unacceptably large anisotropy into the microwave background via gravitational effects.

The strings may be characterized at any time by the mean length of string per unit volume l. For a random (Brownian) configuration with persistence length ξ, one has $l \approx \xi^{-2}$. If μ is the mass per unit length, or tension, we require

$$\mu l \ll \rho. \tag{6}$$

It should be noted that if the configuration of strings simply expands conformally with the expansion of the Universe then

$$l \propto R^{-2},$$

where as ρ scales like R^{-3} or R^{-4}. Thus strings would soon come to dominate unless there were some mechanism to reduce l and transfer energy from strings to other matter or radiation.

However there are such mechanisms. In the very early stages, strings are heavily damped by interaction with surrounding matter, but this condition does not persist for long (Kibble 1976, 1982). Thereafter the most important mechanism appears to involve the formation of closed loops. When strings intersect they may exchange partners. Sometimes this process yields closed loops which then oscillate, gradually losing energy by gravitational (or perhaps other) radiation until they disappear (Vilenkin 1981; Kibble & Turok 1982; Bhattacharjee *et al.* 1982). It is not easy to estimate the rate of production of closed loops, but simple scaling arguments suggest that it should be sufficient to ensure that (after an initial period) the typical length scale ξ of strings remains a constant fraction of the horizon distance. This means that

$$l \approx \xi^{-2} \approx t^{-2}, \tag{7}$$

so that the ratio $\mu l/\rho$ is in fact approximately constant, and compatible with (6). It is possible that the small loops that survive for long periods may be of relevance to the problem of galaxy formation (Vilenkin 1981; Kibble & Turok 1982; Turok 1983).

Fluctuations

Let us now turn to the other type of inhomogeneity: spatial fluctuations in the equilibrium parameters.

It is customary (Peebles 1980; see also Bardeen 1980) to classify the small perturbations on the Friedman–Robertson–Walker universes in three classes: transverse gravitational waves, adiabatic perturbations (i.e. spatial fluctuations in temperature and density with constant baryon-to-entropy ratio) and isothermal fluctuations (spatial fluctuations on the baryon-to-entropy ratio at constant temperature). In fact there might be several sorts of isothermal fluctuations, because in principle there might be independent fluctuations in the three lepton-to-entropy ratios.

Gravitational waves are simply another type of radiation which, because they decouple rather early, seems not to be of great cosmological significance. I shall not consider them further.

One of the important predictions made by the standard GUT scenario for baryon-number generation is that there should be essentially no isothermal perturbations. This is because the baryon-number-generating mechanism depends only on the temperature and so will always yield the same baryon-number density at a given temperature. It would only be possible to escape this conclusion if we found a way to make the maximum reheating temperature vary from place to place, or, as Stephen Hawking pointed out to me, if the CP-violating parameter were variable.

The new inflationary universe also makes very definite predictions about the spectrum of adiabatic perturbations. It is convenient to characterize the magnitude of the perturbation by the value of the density contrast $\delta\rho/\rho$ at the moment when it comes within the horizon, $(\delta\rho/\rho)_h$, say. It has been shown by several different groups that the new inflationary universe generates a spectrum of perturbations for which $(\delta\rho/\rho)_h$ is almost constant, independent of wavelength (Hawking 1982; Albrecht & Steinhardt 1982; Starobinsky 1982; Vilenkin & Ford 1982; Linde 1982*b*; Guth & Pi 1982). This is very encouraging, because this is exactly the spectrum required by the Zel'dovich 'pancake' theory of galaxy formation (Zel'dovich 1970; Harrison 1970).

Unfortunately, though the shape is right the magnitude is substantially too large. However, several authors have suggested that this problem may be avoided in a supersymmetric inflationary model.

Albrecht *et al.* (1982, see also Banks & Kaplunovsky 1982; Pi 1982) proposed a model incorporating the O'Raiffeartaigh type of supersymmetry breaking (O'Raiffeartaigh 1975), as in the reverse-hierarchy mechanism (Witten 1981) or the geometric hierarchy (Dimopoulos & Raby 1983). This naturally yields a very slowly-varying potential, and because the inflation occurs away from $\phi = 0$ the fluctuations may be reduced in magnitude. However, for this type of symmetry breaking the minimum of the (positive-definite) tree-level potential occurs at a non-zero value, and it may therefore be difficult to reconcile with a naturally vanishing cosmological constant. It is also difficult to ensure adequate reheating to make baryons.

A rather different supersymmetric inflationary model due to Ellis *et al.* (1982) envisages a transition temperature very close to the Planck mass (see also Nanopoulos *et al.* 1983). This model is expected to yield acceptably small perturbations (Ellis *et al.* 1983), but on the other hand fails to solve the monopole problem.

Conclusions and discussion

Let us recall the various parameters that are required to fix the initial state at 1 ps and see how many of them can be understood or predicted by the model of phase transitions, and particarly the new inflationary universe.

The near-vanishing of the cosmological constant is still unexplained, though it may perhaps be easier to explain in the context of a supergravity model (Hawking 1983).

The inflationary universe naturally explains the small value of $(\rho-\rho_c)/\rho$, and indeed makes a definite prediction that within observational limits ρ should be exactly equal to ρ_c. It also predicts near-zero values for the densities of defects of all kinds that are produced at or before the inflationary transition, though it leaves open the possibility that strings (or perhaps monopoles) might be produced at a later transition.

There is a qualitative understanding of the baryon-to-entropy ratio, and a clear prediction that the lepton-to-baryon ratio should be unity.

The new inflationary universe naturally explains the overall homogeneity of the Universe, since all we can now observe has evolved from one single tiny bubble. So far as fluctuations are concerned, the predicted spectrum of adiabatic fluctuations has the right shape. The best hope of producing the right magnitude too seems to lie with supersymmetric models. In general one would not expect any isothermal fluctuations.

One thing I have not discussed is the state of the Universe *before* the inflationary phase transition. In fact we know very little about it. It is characteristic of inflation that it erases almost all information about the pre-existing state. The Universe may have been extremely homogeneous or extremely inhomogeneous, but since our observations are confined to what was the interior of a single tiny bubble, we have no means of knowing. It might also be that the transition occurred not by formation of a bubble but instead that the Universe as a whole made a transition to the new phase (Hawking & Moss 1982). Even more exotically, it has been suggested (Vilenkin 1982) that what existed before the transition was nothing at all, that our Universe was created literally from nothing! Though one can indeed perform a quantum-mechanical tunnelling-probability calculation for this conjectured process it is very hard to know how to interpret it!

In any event we cannot expect to go much further back in time without encountering quantum gravity effects (at around the Planck mass), and since there is as yet no wholly satisfactory theory of quantum gravity this is at present impossible.

What is remarkable, however, is the success of the new inflationary universe in explaining so many of the parameters that describe our Universe. This must give good ground for confidence that we are least on the right lines.

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Discussion

S. Kasdan (*Imperial College, Prince Consort Road, London SW7 2BZ, U.K.*). There is now a growing body of literature – by a growing body I mean more than two papers – that establishes that the effective potential, $V(\phi)$, does not exist as a calculable quantity for a range of ϕ around the origin when symmetry breaking takes place and ϕ acquires a non-zero vacuum expectation value. Results by Callaway & Maloof (1983) and Haymaker & Perez-Mercader (1983) show that possible broken symmetry phases cannot be compared by calculating $V(\phi)$ because $V(\phi)$ is actually undefined in the region of interest. Therefore, one cannot determine, say, the true vacuum by an apparently calculable lowest minimum of $V(\phi)$.

T. W. B. Kibble. I accept what Dr Kasdan says, but I regard that as an essentially technical problem. I would be surprised if it affected the physical results.

S. Kasdan. It is more than a technical problem. One is attempting to determine the existence and direction of possible phase transitions by looking at the energy differences between minima of $V(\phi)$ and these results (Callaway & Maloof 1983; Haymaker & Perez-Mercader 1983) imply that this is really a meaningless or undecidable question. It is interesting that this problem was actually first studied in perturbation theory by Coleman & Weinberg (1973) when they developed effective potential methods and it has been ignored by everyone since then. Coleman & Weinberg

(1973) observed that when ϕ developed a non-trivial minimum at $\phi = \Lambda$, their loop expansion failed for $\phi \leqslant \Lambda$. Therefore, the position, depth and even the existence of the minimum could not be trusted. In fact, some years earlier, Symanzik (1970) constructed a proof that $V(\phi)$ is always concave upwards. So, these new results (Callaway & Maloof 1983; Haymaker & Perez-Mercader 1983) show that the difficulty with $V(\phi)$ is not an artefact of Coleman & Weinberg's methods and they agree with the general result of Symanzik.

There may be some other way to do what Professor Kibble wants to do, but calculating $V(\phi)$ does not seem to be the way. Questions as to true and false vacua possibly cannot be decided purely within the context of quantum field theory and some additional physics might be needed.

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